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THREE-DIMENSIONAL DISTURBANCES IN HIGH SPEED BOUNDARY LAYER FLO--ETC(U)
AUG 81 G R INGER F49620-76-C-0013
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report summarizes developments on a new theory which was devised to identify and describe the cause of the pronounced spanwise-periodic disturbances observed in reattaching separated flows. This theory was subsequently extended to include surface mass transfer effects and the coupling of the boundary layer disturbances and adjacent surface material thermal and ablative response of the body surface. It was shown for the first time that a similar three-dimensional phenomenon can occur near the separation line of a high speed boundary layer. In parallel, the theoretical study of reattachment-

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) stagnation region 3-D vortices which was refined using a systematic inner-outer match asymptotic analysis approach appropriate to finite-sized bodies is described. A very careful detailed formulation of the associated boundary value problem for the case of a cylinder and its detailed numerical treatment (which also required great effort and care) has nearly been completed. Finally, a new turbulent viscous-disturbance sublayer theory which was developed as the inner deck of a generalized non-asymptotic triple-deck theory of two-dimensional turbulent shock interaction problems in both transonic and supersonic flows is discussed. This theory provides a much improved theoretical account of the skin friction, upstream influence and displacement thickness growth in a wide variety of such problems.

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# Three-Dimensional Disturbances in High Speed Boundary Layer Flows

FINAL SCIENTIFIC REPORT

AFOSR CONTRACT F49620-76-C-0013

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Blacksburg, Va. 24061



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## 1. Introduction

The objective of this research is the basic theoretical investigation of three-dimensional pressure, skin friction and heat transfer disturbances in both laminar and turbulent boundary layer flows including viscous-inviscid interaction effects, separation and reattachment. A sound understanding of these phenomena is required in modern aerodynamic design analyses of high speed flight vehicles.

The aforementioned need has led to a systematic basic research effort treating in parallel the twin problems of (1) viscous-inviscid interaction with flow separation and/or attachment and (2) three-dimensional effects (including those due to streamwise vortices) in high speed laminar and turbulent boundary layer flows along two-dimensional and axi-symmetric bodies, with emphasis on pressure distribution, skin friction and heat transfer effects. Such a study was initiated October 1, 1971, and has continued under AFOSR support. To date, it has been very successful both in solving a number of the target flow problems and in developing new ideas and tools for attacking new aspects of (1) and (2). This report summarizes these research results.

## 2. Summary of Technical Accomplishments

A new theory was devised[1] which identified and described the cause of the pronounced spanwise-periodic disturbances observed[2] in reattaching separated flows (see Figure 1). This theory was subsequently extended[3] to include surface mass transfer effects and the coupling of the boundary layer disturbances and adjacent surface material thermal and ablative response of the body surface (Figure 2). Furthermore, it was shown[4] for the first time that a similar three-dimensional phenomenon can occur near the separation line of a high speed boundary layer (Figure 3). During the author's Fullbright-von Humboldt Research Fellowship at the DFVLR-AVA in Göttingen, West Germany, an improved theory of two-dimensional separation and reattachment regions of either laminar or turbulent flow was also developed[5,6] to determine the streamline angle and flow curvature therein, both of which are very important to the 3-D disturbance study (Figure 4). These separation-region theories were further subjected to verifying experimental comparisons with data (see Figure 5) supplied by Professor Bogdonoff's group at Princeton[11], with whom active collaboration has been established. In parallel, the theoretical study of reattachment-stagnation region 3-D vortices was refined using a systematic inner-outer matched asymptotic analysis approach appropriate to finite-sized bodies. A very careful detailed formulation of the associated boundary value problem for the case of a cylinder and its detailed numerical treatment (which also required great effort and care) was nearly finished; the goal was to predict a unique spanwise wavelength for these disturbances as experimentally observed. Informal collaboration on this subject was established with Professors Mark Morkovin of the Illinois Institute of Technology (Figure 6) and W. Sadeh of Colorado State University.

Turning to work on viscous-inviscid interactions, several fruitful developments occurred. First, a new turbulent viscous-disturbance sublayer theory was developed as the inner deck of a generalized non-asymptotic triple-deck theory[12] of two-dimensional turbulent shock interaction problems in both transonic and supersonic flows (Figure 7). This theory provides a much improved theoretical account of the skin friction, upstream influence and displacement thickness growth in a wide variety of such problems. In particular, it provides the heretofore-missing link between Lighthill's theory for lower Reynolds numbers and modern asymptotic theory pertaining to the infinite Reynolds number limit (Figure 8). Second, somewhat as a fortuitous side development, some fundamental studies[13,14] were carried out on how vectored suction or injection through the surface influences laminar boundary layer viscous-inviscid flow properties; one of the major results was the discovery of some new branches of the family of self-similar solutions to the laminar boundary layer equations. Third, the author spent three months (6/16-9/18) as a visiting researcher at the Princeton University Gas Dynamics Laboratory for the purpose of developing a basic analytical framework for certain Princeton experimental studies of three-dimensional shock-boundary layer interactions. Princeton's experimental approach offers two important advantages to the theoretician: (a) the physical configurations are simple geometries offering the best opportunity for fundamental diagnosis; (b) the measurements involve a systematic variation of the interaction strength over a wide range including the weaker non-separating regime. A detailed study of various theses and papers describing their results (plus many of the references therein) indicated that the most promising problem for initial attack was the sharp vertical fin study by Oskam[15] (Figure 8). A mathematically well-posed analytical model of this 3-D turbulent interaction field was formulated for fin deflection angles below that causing separation, and a solution approach then devised using operational methods. This theory constitutes a three-dimensional generalization of the author's aforementioned non-asymptotic turbulent triple-deck theory for 2-D interactions.[12] In Appendix A we show some of the essential steps involved in setting up the solution approach to this vertical fin problem, based on illustrations from a lecture presented by the author at Princeton in September 1980. Although considerable detailed work is still required to complete the solution, the sought-after foundation was established; the ultimate results are expected to shed valuable light on (among other things) upstream and lateral influence properties, skin friction line geometry and incipient separation, and deviations from cylindrically-symmetric behavior.

## 3. Summary of Publications and Presented Papers on Behalf of This Contract

Overall this research effort produced nine major papers (Refs. 1, 3, 4, 5, 6, 7, 12, 13, 14) of which six have been published so far in the open literature (Refs. 1, 3, 5, 6, 13, 14) and seven were presented at major technical meetings (Refs. 1, 3, 6, 7, 12, 13, 14).

In addition, seminars on selected aspects of the research were presented at various times throughout the contract period at the University of Colorado, Johns Hopkins University Applied Physics Laboratory, University of Arizona, Princeton University (as a Baetjer Lecturer), University of Delaware, State University of New York at Buffalo, the von Karman Institute (Brussels), University of Trondheim (Norway), Université de Poitiers (France), and the University of Toronto (at the 1980 IUTAM Meeting).

## 4. Educational Impact of the Research

Various phases of this work have formed the graduate training of three Ph.D. students (R. C. Swanson and T. F. Swean of VPI&SU plus M. Namtu of Romania) and two M.S.-level students (K. M. Cho, subsequent Ph.D. candidate at Cornell and V. Sonnad, subsequent Ph.D. candidate at M.I.T.).

## References

1. Inger, G. R., "Three-Dimensional Disturbances in Reattaching Separated Flows," Proc. AGARD Symposium on Flow Separation, CP-168, pp. 18-1 to 18-12, Göttingen, May 1975.

2. Ginoux, J., "The Existence of Three-Dimensional Perturbations in the Reattachment of a Two-Dimensional Supersonic Boundary Layer After Separation," AGARD Report 272, April 1960.

3. Inger, G. R., "Three-Dimensional Heat and Mass Transfer Effects Across High Speed Reattaching Flows," VPI&SU Report Aero-036, Blacksburg, Virginia, July 1975. Presented as AIAA Paper 76-160, 14th Aerospace Sciences Meeting, Washington, D.C., January 1976; AIAA Journal 15, March 1977, pp. 383-389.

4. Namtu, M., and Inger, G. R., "Spanwise-Periodic Three-Dimensional Disturbances in Nominally 2-D Separating Laminar Boundary Layer Flows," VPI&SU Report Aero-052, Blacksburg, Virginia, August 1976.

5. Inger, G. R., "On the Curvature of Compressible Boundary Layer Flows Near Separation," Zeitschrift für Agenwante Mekanik und Physik ("ZAMP"), Vol. 28/6, 1977, pp. 1027-1035.

6. Inger, G. R., "Compressible Boundary Layer Flows Near Separation or Reattachment," Proc. 1978 Heat Transfer and Fluid Mech. Inst., Stanford U. Press.

7. Inger, G. R., "Compressible Laminar and Turbulent Boundary Layers Near Separation or Reattachment," AIAA Paper 78-1164, 11th Fluid and Plasma Dynamics Conf., Seattle, July 1978.

8. Roshko, A., and Thomke, G. J., "Observations of Turbulent Reattachment Behind an Axi-symmetric Downstream Facing Step in Supersonic Flow," AIAA Journal 4, June 1966.

9. Ginoux, J., "Effect of Mach Number on Streamwise Vortices in Laminar Reattaching Flows," VKI TN 26, July 1965.

10. Miller, D. S., et al., "Mach 8 to 22 Studies of Flow Separation Due to Deflected Control Surfaces," AIAA Journal 2, February 1964.

11. Settles, G. S., Fitzpatrick, T. J., and Bogdonoff, S. M., "A Detailed Study of Attached and Separated Compression Corner Flowfields in High Reynolds Number Supersonic Flow," AIAA Paper 78-1167, July 1978.

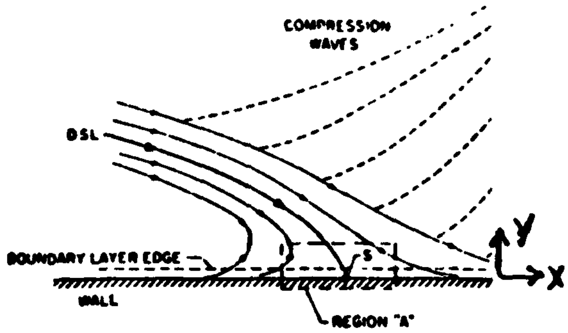
12. Inger, G. R., "The Skin Friction-Disturbance Sublayer in Non-separating Shock-Turbulent Boundary Layer Interactions," AIAA Paper 80-1411, 13th AIAA Fluid and Plasma Dynamics Conf., Snowmass, Colorado, July 1980.

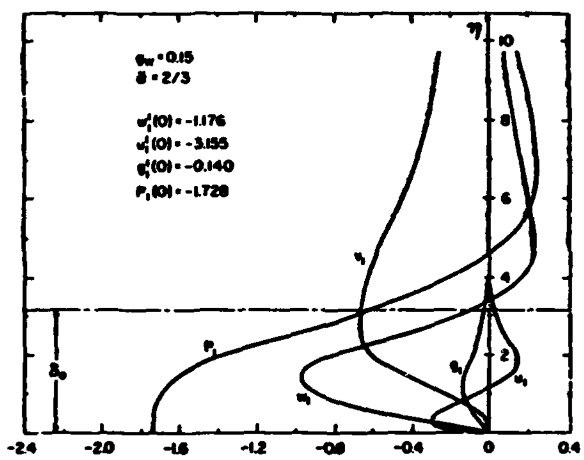
13. Inger, G. R., and Swean, T. F., "Vectored Injection into Laminar Boundary Layers with Heat Transfer." Presented as Paper 74-676, AIAA/ASME 1974

Thermophysics Conf., Boston, July 1974; AIAA Journal 13, May 1975, pp. 616-622.

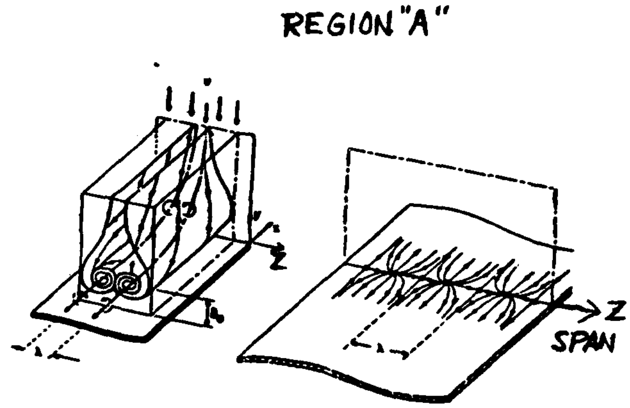
14. Inger, G. R., and Swean, T. F., "Hypersonic Viscous-Inviscid Interaction with Vectored Surface Mass Transfer." Presented as Paper 75-84, AIAA 8th Fluid and Plasma Dynamics Conf., Hartford, June 1975; AIAA Journal 14, March 1976, pp. 589-596.

15. Oskam, B., "Three-Dimensional Flowfields Generated by the Interaction of a Swept Shock Wave with a Turbulent Boundary Layer," Ph.D. Thesis, MAE Department, Princeton University, 1976.

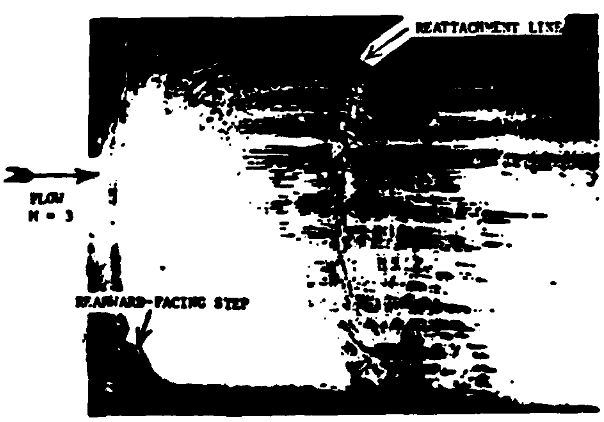



Eigenvalue Disturbance Profiles Across Boundary Layer

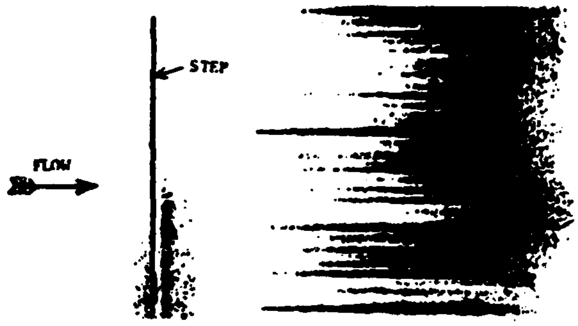



Vortex-Instability Disturbance Flow (Schematic)

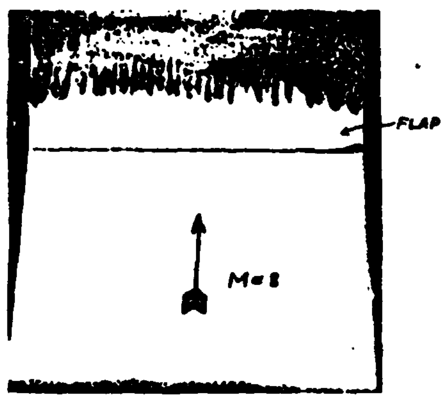



Experimental Oil Flow Visualization of Reattachment Flow Pattern[8]

Figure 1. Spanwise-Periodic Disturbance-Vortices in Reattaching Shear Flows (Ref. 1)



Typical surface thermal response (subliming film [4]



Spanwise scorching pattern in hypersonic reattachment region (steel flap [10]

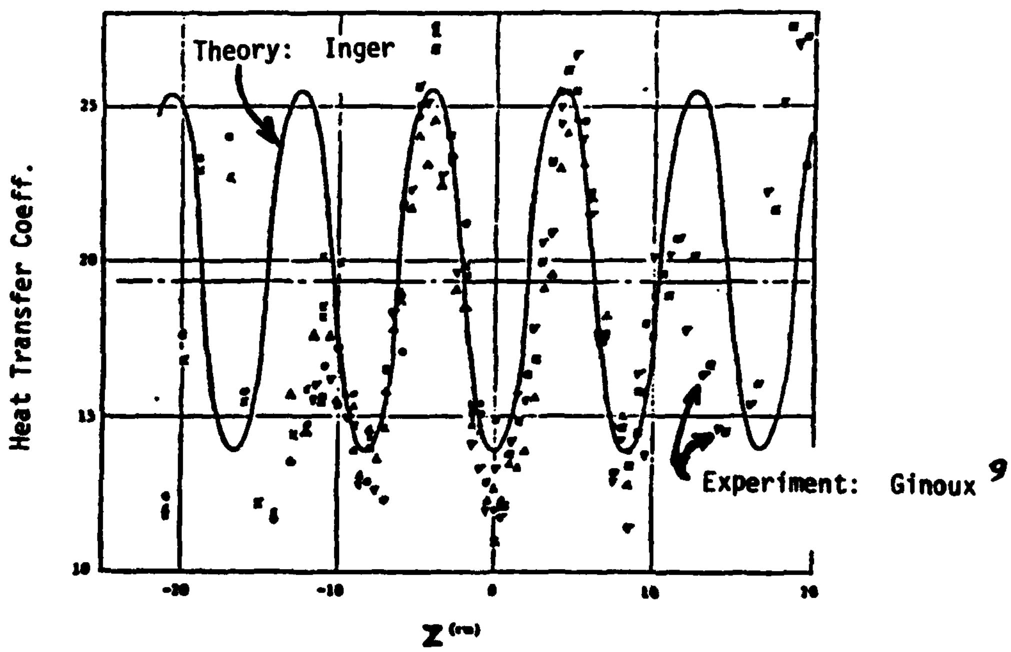



Figure 2. Heat Transfer and Surface-Material Response Aspects of the Spanwise-Disturbance Vortices in Reattaching Flows

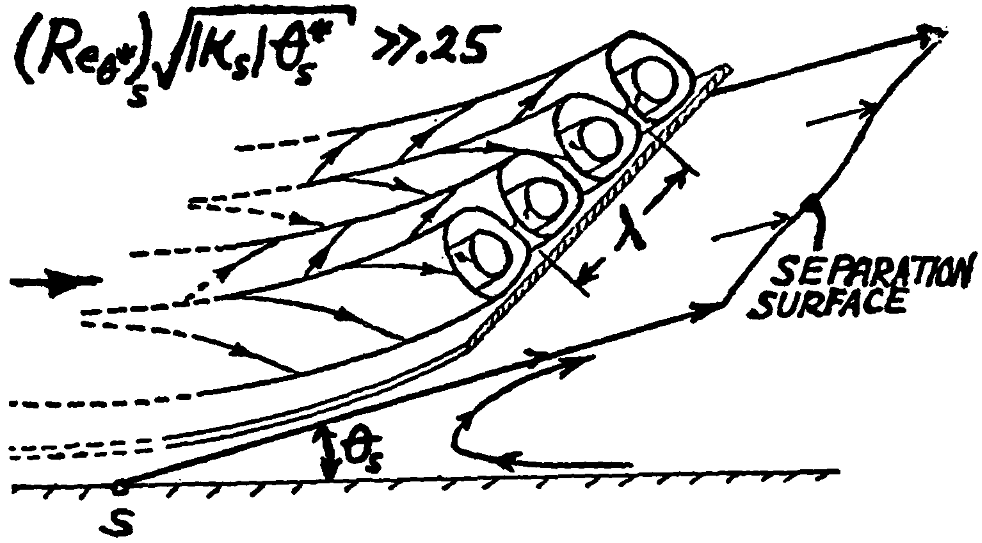
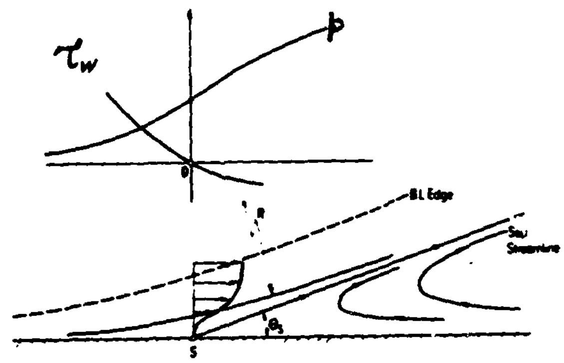


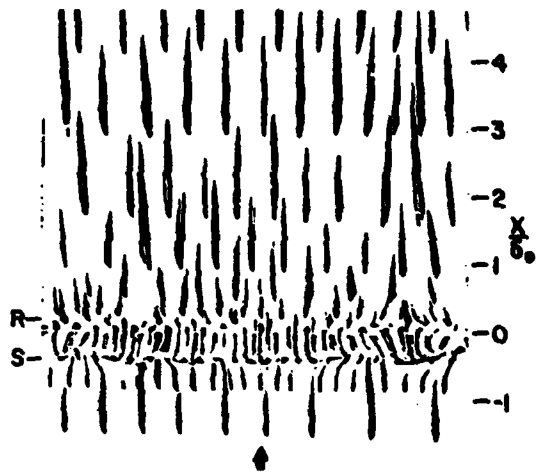

Figure 3. Streamwise Vortex-Disturbances in Separating Flows (Ref. 4)




$$\tan \theta_s \simeq -3 \, (\partial \tau_w / \partial x)_s \, / \, (\partial p / \partial x)_s$$

$$R \simeq - \frac{3 (\partial \tau_w / \partial x)_s}{(\partial^2 \tau_w / \partial x^2)_s} \cdot \cot \theta_s \simeq \frac{(\partial p / \partial x)_s}{(\partial^2 \tau_w / \partial x^2)_s}$$

Figure 4. Separation-Neighborhood Analysis for Two-Dimensional Laminar or Turbulent Flows (Ref. 6,7)




A.
Surface Streak Flow Pattern

B.
Separation Line Detail

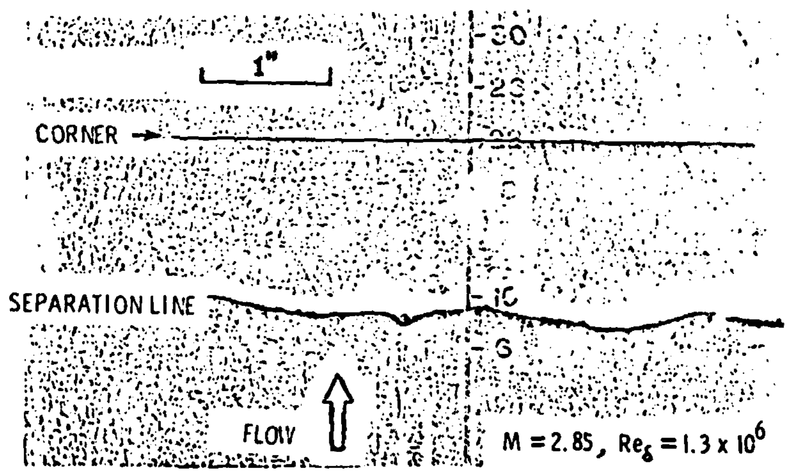



Figure 5. Experimental Confirmation of Theoretically-Predicted Separation-Region Vortices (Ref. 11)

NASA Contractor Report 3231

# On the Question of Instabilities Upstream of Cylindrical Bodies

Mark V. Morkovin
*Illinois Institute of Technology*
*Chicago, Illinois*

" The rather unconventional views presented here in a searching spirit were first written up in May 1976. Eighteen copies of that report were circulated in the United States, Europe, and Japan to elicit critique and suggestions and to provide an opportunity for private prepublication rebuttals for those whose earlier work was questioned in the text. The present report incorporates, then, not only the information and views from publications since 1976 but also the advice and comments of a number of discussors, in particular Profs. G. Inger, E. Roshotko, and I. Tani. Figures based on the reviewed publications are explicitly credited. The original NASA study under Grant NSG-1120 was guided, with understanding, by Ivan Beckwith. The 1979 revision was supported under AFOSR Contract F49620-77-C-013 concerned with instabilities and transition to turbulence. "

Figure 6.

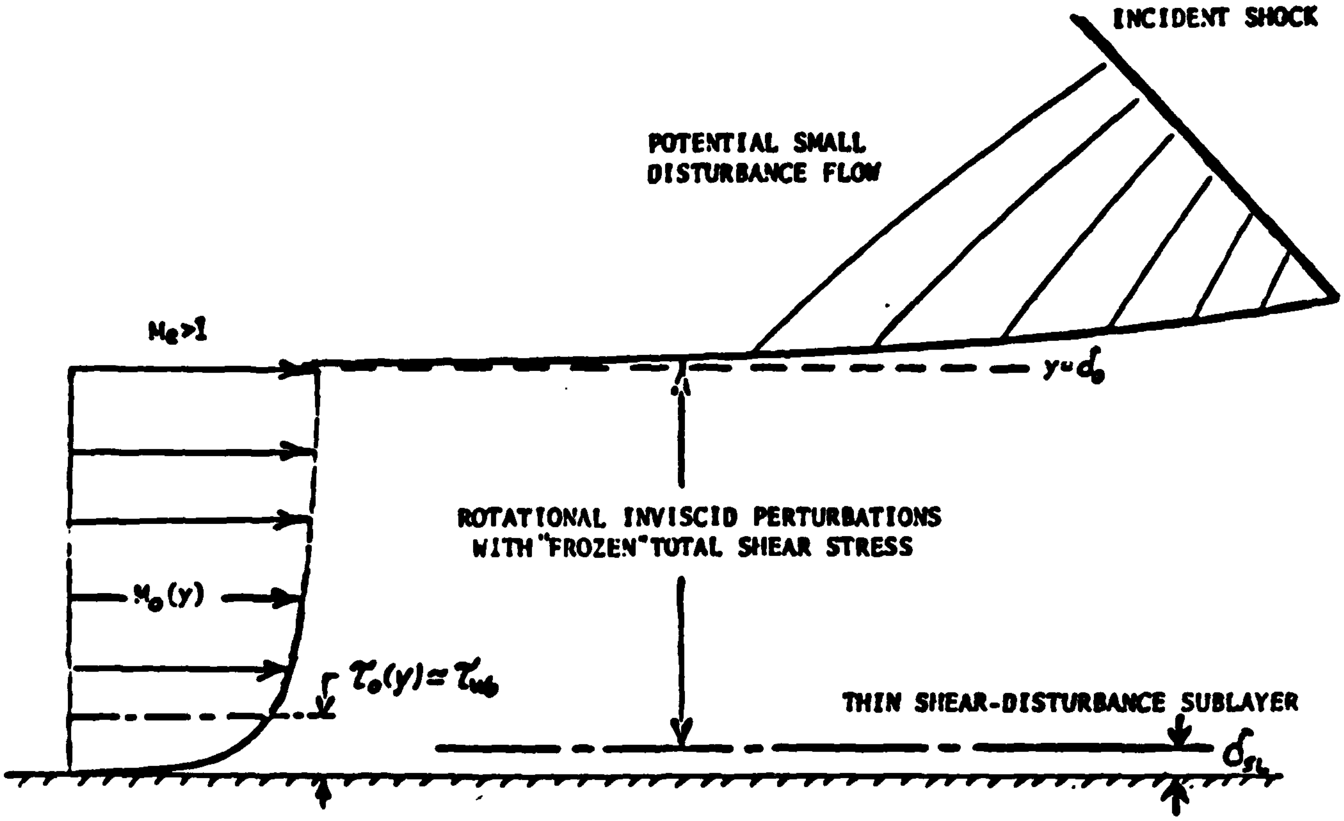



Figure 7. Non-Asymptotic Triple-Deck Model of Shock-Turbulent Boundary Layer Interactions (Ref. 12)

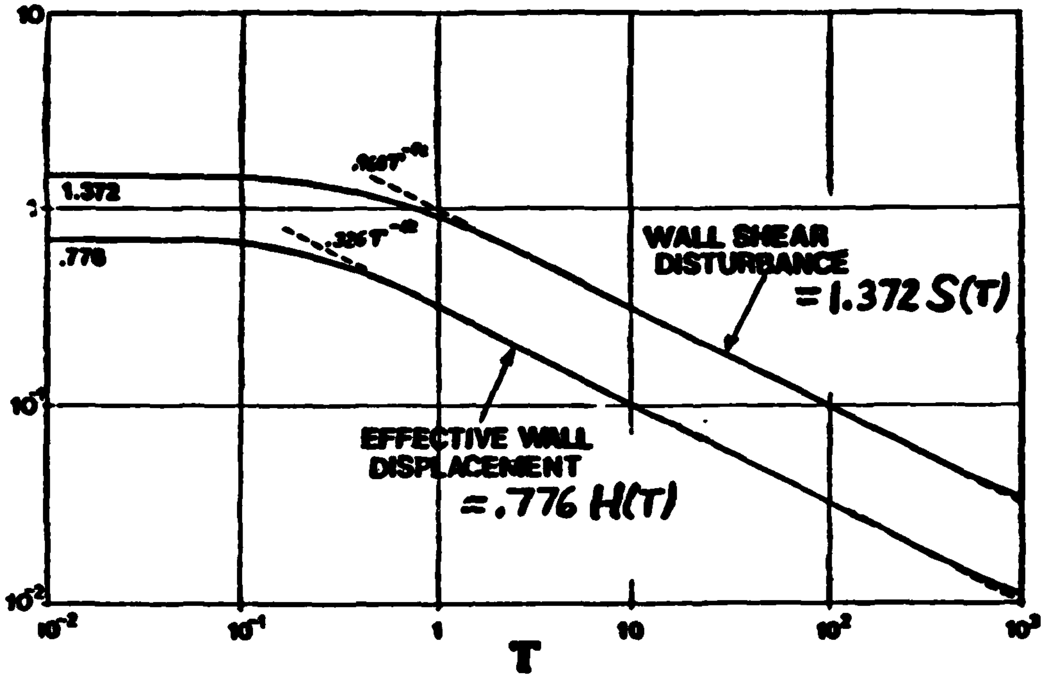



Figure 8. Effect of Interactive Turbulence Parameter T on Inner Deck Properties (from Ref. 12)

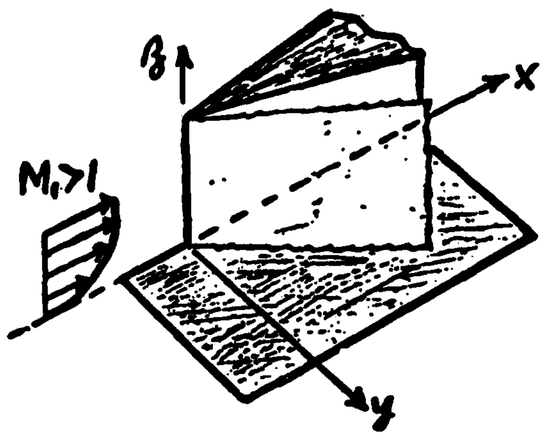
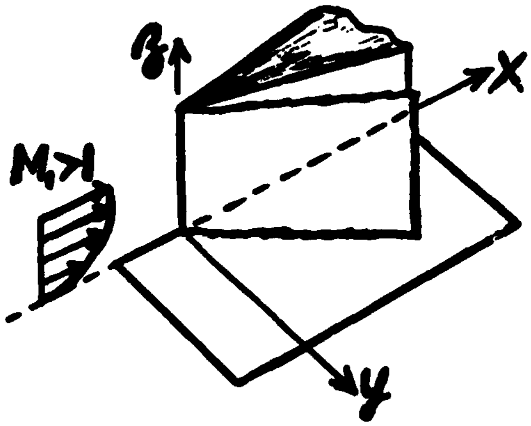



Figure 9. The Slender Sharp Vertical Fin-Boundary Layer Interaction Problem (Schematic)

# Appendix A

## Details of the 3-D Triple-Deck Analysis of the Vertical Fin Shock-Boundary Layer Interaction Problem




$$\frac{\partial \rho'}{\partial x} U_0 + \rho_0\left(\frac{\partial u'}{\partial x} + \underline{\frac{\partial v'}{\partial y}} + \frac{\partial w'}{\partial z}\right) + w' \frac{d\rho_0}{dz} = 0 \qquad \text{CONTINUITY}$$

$$U_0 \frac{\partial u'}{\partial x} + w' \frac{dU_0}{dz} + \frac{1}{\rho_0}\frac{\partial p'}{\partial x} \simeq 0 \qquad \text{STREAMWISE MOMENTUM}$$

$$U_0 \frac{\partial w'}{\partial x} + \frac{1}{\rho_0}\frac{\partial p'}{\partial z} \simeq 0 \qquad \text{VERTICAL MOMENTUM}$$

$$U_0 \frac{\partial v'}{\partial x} + \frac{1}{\rho_0}\frac{\partial p'}{\partial y} \simeq 0 \qquad \underline{\text{LATERAL MOMENTUM}}$$

$$U_0 \frac{\partial p'}{\partial x} \simeq a_0^2\left(U_0 \frac{\partial \rho'}{\partial x} + w' \frac{d\rho_0}{dz}\right) \qquad \text{ENERGY}$$

Notes:
- ADIABATIC
- FROZEN TOTAL SHEAR STRESS
- "PARALLEL" UNDISTURBED FLOW ($V_0 \simeq \frac{\partial U_0}{\partial x} = 0$)

GOVERNING 3-D DISTURBANCE EQUATIONS FOR INVISCID OUTER AND MIDDLE DECK REGIONS

- INCOMPRESSIBLE DISTURBANCE MOTION
- VERY THIN LAYER, $\partial p'/\partial z = 0$ & $\rho_0 \approx \rho_{0w}$

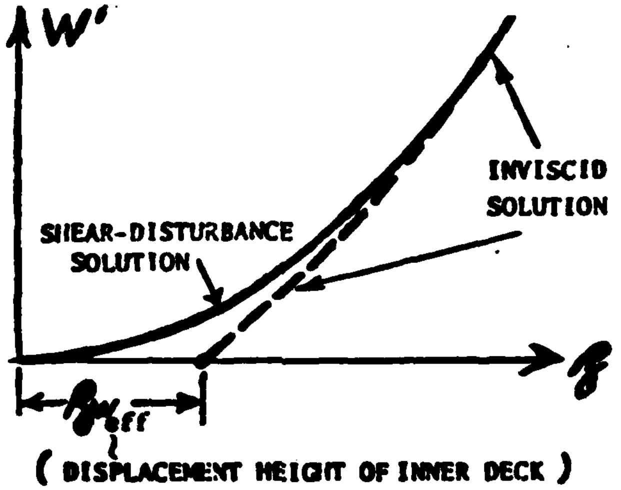
$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0$$

$$U_0(z)\frac{\partial u'}{\partial x} + \frac{dU_0}{dz}w' + \frac{1}{\rho_w}\frac{\partial p'}{\partial x} = \nu_w\left(\frac{\partial^2 u'}{\partial z^2} + \underline{\frac{\partial^2 u'}{\partial y^2}} + \frac{\partial^2 u'}{\partial x^2}\right)$$

$$U_0(z)\frac{\partial v'}{\partial x} + \frac{1}{\rho_{0w}}\frac{\partial p'}{\partial y} = \nu_{0w}\left(\frac{\partial^2 v'}{\partial z^2} + \underline{\frac{\partial^2 v'}{\partial y^2}} + \frac{\partial^2 v'}{\partial x^2}\right)$$

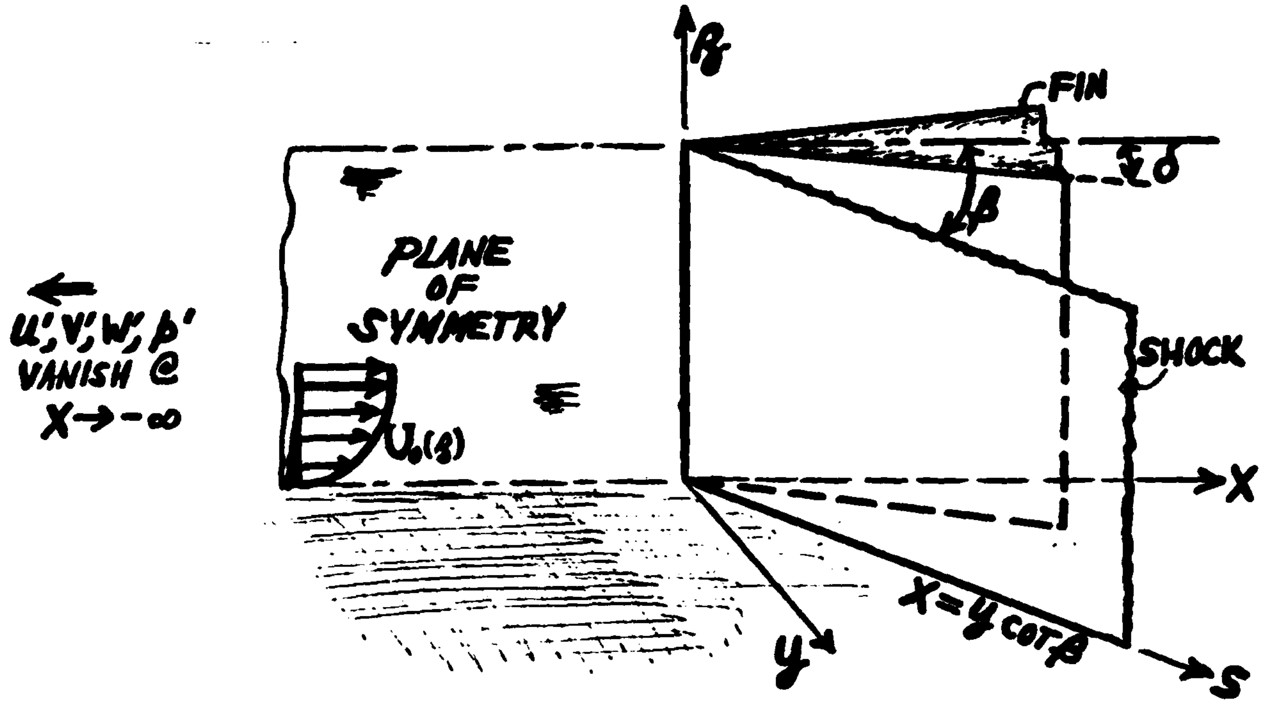
---

$$\frac{\partial}{\partial x}\left(U_0\frac{\partial^2 w'}{\partial z^2} - \frac{d^2 U_0}{dz^2}w'\right) = \nu_{0w}\frac{\partial^2}{\partial z^2}\left(\frac{\partial^2 w'}{\partial z^2} + \underline{\frac{\partial^2 w'}{\partial y^2}} + \frac{\partial^2 w'}{\partial x^2}\right)$$




**GENERAL 3-D VISCOUS DISTURBANCE PROBLEM for INNER DECK**

.RY
IN I




INNER LATERAL ($y = 0$)

$X < 0$ (PLANE OF SYMMETRY):

$$\partial p'/\partial y = 0$$
$$v' = 0$$
$$w' \simeq 0$$ (NEGLIGABLE UPWASH FOR SHARP FIN)

$X \geq 0$ (FIN FACE): $v'(x, 0, z) \simeq U_0(z)\delta$

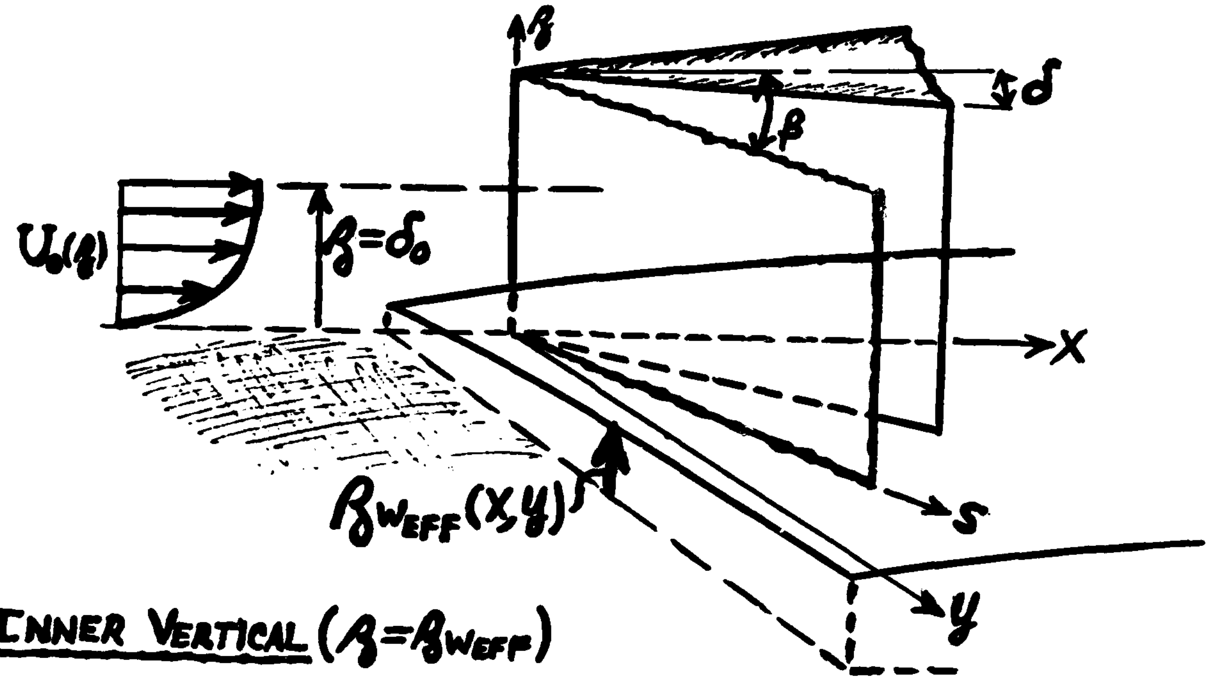
$$\left[\partial p'/\partial y = -\rho_0(z) U_0(z) \frac{\partial v'}{\partial x}\right]$$

OUTER LATERAL ($y \to \infty$)

DISTURBANCE FIELD BECOMES CYL.-SYMM., $\frac{\partial}{\partial s} \to 0$:

$$\frac{\partial p'}{\partial y}(x, \infty, z) + \cot\beta \cdot \frac{\partial p'}{\partial x}(x, \infty, z) = 0$$

INVISCID BOUNDARY CONDITIONS FOR VERTICAL FIN PROBLEM




INNER VERTICAL ($z=z_{w_{EFF}}$)

$$w' = \frac{\partial p'}{\partial z} = 0$$

MIDDLE – OUTER DECK MATCHING ($z=\delta_0$)

$p'$ and $w'$ (hence $\frac{\partial p'}{\partial z}$) continuous

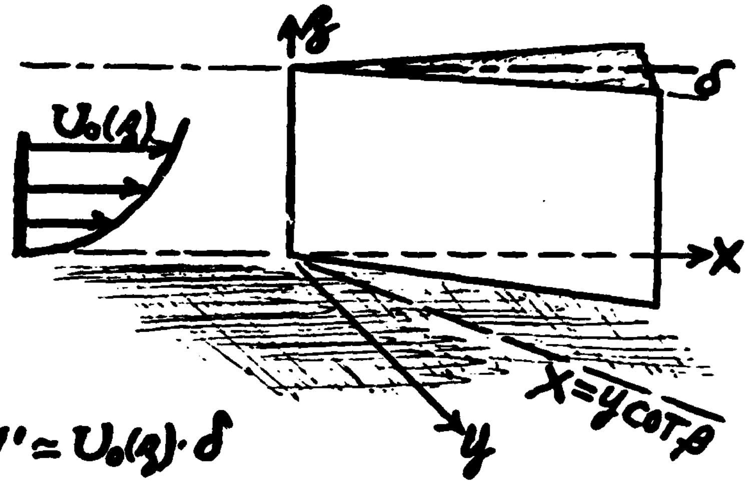
FAR FIELD VERTICAL ($z\to\infty$)

$$p' = 0\ ,\quad X < y\cot\beta$$
$$= \gamma p_1 M_1^2\, \delta / \sqrt{M_1^2-1}\ ,\quad X \geq y\cot\beta$$

(UNIFORM PRESSURE DISTURB. DUE TO FIN $\delta$)

(NOTE : ENFORCING THIS CONDITION AUTOMATICALLY INSURES SATISFACTION OF $\partial p'/\partial s \to 0$ AT $y\to\infty$)

INVISCID BOUNDARY CONDITIONS (CONTINUED)




**INNER LATERAL** ($y = 0$)

$X < 0$ (PLANE OF SYMM.): $V' = \dfrac{\partial U'}{\partial y} = 0$

$W' = 0$

$X \geq 0$ (FIN FACE): $V' \simeq U_0(z)\cdot\delta$

$W' = 0$ (NO SLIP)

$U' = -U_0(z)$ (NO SLIP)

**OUTER LATERAL** ($y \to \infty$)

$\partial(\,)'/\partial s \to 0$ $\left\{\text{AUTOMATIC SINCE DRIVING } p'(x,y) \text{ SOLUTION HAS THIS PROPERTY}\right.$

**INNER VERTICAL ON FLOOR** ($z = 0$)

$W' = 0$ (IMPERMEABLE & FLAT)

$V' = U' = \dfrac{\partial W'}{\partial z} = 0$ (NO SLIP & FLAT)

$$\mu_w\left[\frac{\partial^3 W'}{\partial z^3}(0) + \frac{\partial^3 W'}{\partial y^2 \partial z}(0)\right] = -\left(\frac{\partial^2 p'}{\partial x^2} + \frac{\partial^2 p'}{\partial y^2}\right) \quad \left\{\text{"COMPATIBILITY" EQN}\right.$$

**OUTER VERTICAL** ($z \gg \delta_{SL}$)

$U', V', W' \to U'_{INV}, V'_{INV}, W'_{INV}$ $\left\{\begin{array}{l}\text{INVISCID}\\ \text{FORM OF}\\ X, Y \text{ MOM. EQNS}\end{array}\right\}$

# VISCOUS DISTURBANCE FLOW BOUNDARY CONDITIONS IN INNER DECK

STEP 1 : FOURIER TRANSFORM W.R.T. X

$$\tilde{P}(K,y,\zeta) \equiv \int_{-\infty}^{\infty} e^{-iKX} p'(X,y,\zeta)\, dX$$

YIELDS

$$K^2(M_1^2-1)\tilde{P} + \frac{\partial^2 \tilde{P}}{\partial y^2} + \frac{\partial^2 \tilde{P}}{\partial \zeta^2} = 0$$

SUBJECT TO

$$\tilde{P}(K,y,\infty) = M_1^2 \delta\, e^{-iKy\cot\beta} / iK\sqrt{M_1^2-1}$$

$$\frac{\partial \tilde{P}}{\partial y}(K,0,\zeta) = -M_1^2 \delta$$

STEP 2 : FOURIER COSINE TRANSFORM W.R.T. y

$$\tilde{\tilde{P}}(K,m,\zeta) \equiv \int_0^{\infty} \tilde{P}(K,y,\zeta) \cdot \cos my\, dy$$

YIELDS $$\left(K^2\cot^2\beta - m^2\right)\tilde{\tilde{P}} + \frac{d^2\tilde{\tilde{P}}}{d\zeta^2} = 0$$

SUBJECT TO $$\tilde{\tilde{P}}(K,m,\infty) = M_1^2\delta / \left[(iK)^2\cot^2\beta + m^2\right]$$

whose solution is

$$\tilde{\tilde{P}}(K,m,\zeta) = \frac{M_1^2 \delta}{(iK)^2\cot^2\beta + m^2} + \tilde{\tilde{P}}(K,m,\delta_0)\, e^{-\lambda(\zeta-\delta_0)}$$

with $\lambda \equiv i\sqrt{K^2\cot^2\beta - m^2}$

$$\frac{d\tilde{\tilde{P}}}{d\zeta} + \lambda\tilde{\tilde{P}} = \frac{-i\delta M_1^2}{\sqrt{K^2\cot^2\beta - m^2}}$$

SOLUTION METHOD for OUTER DECK PRESSURE FIELD

Attack: FOURIER X-TRANSFORM FOLLOWED BY FOURIER COSINE TRANSFORM W.R.T. $y$

THIS YIELDS THE O.D.E. SPLIT BOUNDARY VALUE PROBLEM

$$\left\{\left[M_0^2(z)-1\right]K^2-m^2\right\}\tilde{\tilde{P}}+\frac{d^2\tilde{\tilde{P}}}{dz^2}-\frac{2}{M_0}\frac{dM_0}{dz}\frac{d\tilde{\tilde{P}}}{dz}=$$

$$=\underbrace{-\delta\, M_0^2(z)}_{\text{SIDE DEFLECTION EFFECT OF FIN}}$$

$$\frac{d\tilde{\tilde{P}}}{dz}(K,m,z_{W_{EFF}})=0$$

$$\frac{d\tilde{\tilde{P}}}{dz}(K,m,\delta_0)+i\sqrt{K^2\cot^2\beta-m^2}\cdot\tilde{\tilde{P}}(K,m,\delta_0)=\underbrace{\frac{-i\delta M_1^2}{\sqrt{K^2\cot^2\beta-m^2}}}_{\text{EFFECT OF OVERLYING FIN-INDUCED SHOCK}}$$

SOLUTION CAN BE OBTAINED BY STANDARD METHODS

MIDDLE DECK SOLUTION

Attack: FOURIER X-TRANSFORM FOLLOWED BY FOURIER SINE TRANSFORM

$$\tilde{\tilde{V}}_s(K,m,\zeta) \equiv \int_0^\infty \tilde{V}(K,y,\zeta)\sin my\,dy$$

THIS YIELDS THE VISCOUS SIDEWASH PROBLEM

$$\frac{d^2\tilde{\tilde{V}}_s}{d\zeta^2} - \left[\frac{iKU_0(\zeta)}{\nu_{0w}} + m^2\right]\tilde{\tilde{V}}_s = -m\left[\frac{\tilde{\tilde{P}}_w}{\mu} + \frac{\delta U_0(\zeta)}{iK}\right]$$

"BOUNDARY REGION" TERMS

where

OVERLYING PRESSURE SPECTRUM FROM BOTTOM OF MIDDLE DECK $\quad \tilde{\tilde{P}}_w = \tilde{\tilde{P}}(K,m,\zeta_{W\,EFF})$ $\quad$ ⟨COSINE TRANSFORM⟩

$$\tilde{\tilde{V}}_s(K,m,0) = 0, \quad \tilde{\tilde{V}}_s(K,m,\zeta_\infty) = \text{INVISCID SOLUTION}$$

THE COMPANION UP-WASH PROBLEM IS:

$$iK\left[U_0(\zeta)\frac{d^2\tilde{\tilde{W}}_s}{d\zeta^2} - \frac{d^2U_0}{d\zeta^2}\tilde{\tilde{W}}_s\right] = \nu_{0w}\frac{d^2}{d\zeta^2}\left[\frac{d^2\tilde{\tilde{W}}_s}{d\zeta^2} - m^2\tilde{\tilde{W}}_s\right]$$

subject to $\quad \tilde{\tilde{W}}_s(0) = \dfrac{d\tilde{\tilde{W}}_s}{d\zeta}(0) = 0$

$$\mu_{0w}\frac{d^3\tilde{\tilde{W}}_s}{d\zeta^3}(0) = -\left(\frac{K^2+m^2}{m}\right)\frac{d\tilde{\tilde{P}}_w}{dm} - \frac{m}{2\pi}\int_0^\infty \tilde{\tilde{P}}_w\,dm$$

SOLUTION OF VISCOUS DISTURBANCE FLOW IN THE INNER DECK

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